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RELIABILITY ESTIMATION OF A TWO UNIT PARALLEL SYSTEM

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ABSTRACT

This paper deals with reliability estimation of a system having two parallel operating units subject to inspection and replacement. Whenever any operating unit fails it is inspected to diagnose the possibility of its either repair or replacement. There is a single repair facility to repair the failed units on first come first serve basis. All the failure time distributions are assumed to be negative exponential while as inspection, repair and replacement time distributions are taken to be arbitrarily. Using regenerative point technique system reliability obtained.

KEYWORDS: Reliability, Mean Time to System Failure, Regenerative Point Technique

INTRODUCTION

In this paper we have studied a two unit parallel system under the specific assumption about the inspection and replacement of the failed unit. Some early works on parallel system have been generalized in one direction only. Dhillan *et al*. analyzed human error modeling of parallel and standby redundant systems. Goel *et al*. studied a two (multi-component) unit parallel system with standby and common cause failure. Chandrsekhar *et al.* Have obtained Confidence limits for steady state availability of a parallel system. Singh *et al*. A study on a two unit parallel system with erlangian repair time. Gopalan *et al*. studied availability and reliability of a series parallel system with a single repair facility. Kishan *et al*. analyzed a two-unit parallel system with preventive maintenance. Mogha *et al*. studied a two unit parallel system with correlated lifetimes and repair machine failure. Montaser et al. obtained reliability equivalence of a Parallel System with non-Identical Components. Gupta *et al*. studied reliability and MTTF analysis of a non-repairable parallel redundant complex system under hardware and human failure.

Description of the System:

- System consists of two parallel identical operating units.
- Whenever operating unit fails it goes to inspection. After inspection failed unit either goes to repair or replacement by the new unit.
- After repair, unit does not work as a new unit .it works as a quasi new one.
- 4. There is a single repair facility which repair, inspect and replaced the unit on first come first serve basis.
- All the failure, repair & inspection time distributions follow negative exponential distribution.

Notations:

λ : Constant failure rate of working unit.

λ_1 : Constant failure rate of degraded unit.

r : Repair rate of failed unit.

η : Constant Replacement rate.

δ : Constant Inspection completion rate of failed unit.

p : Probability that failed unit goes to repair.

q : Probability that failed unit goes to replacement by new unit.

[S] : Laplace Stieltjes convolution

© : Laplace convolution

$q_{ij}(t)$,$Q_{ij}(t)$: pdf and cdf from state i to j.

E : Set of regenerative states S_i {i=0-11}

μ_i : Mean sojourn time

$p_{ij:}$ transition probability from state S_i to S_j

$\pi_i(t)$: cdf of time to system failure without time t starting from state S_i.

Symbols for the States of the System:

$N_O / N_O^{'}$: Normal/ Quasi normal unit in operating mode

F_{wI}, F_I, F_r : Failed unit waiting for inspection / under inspection / under repair.

U_d : Failed unit is under replacement.

Possible transitions between states are shown in Figure.

Transition Probabilities and Sojourn Times:

Simple probabilistic considerations yield the following expressions for transition probabilities p_{ij}:

$$P = (p_{ij}) = [Q_{ij}(\infty)] = Q(\infty)$$

$$p_{01} = 1;\ \ p_{12} = \frac{p\delta}{A_1};\ \ p_{13} = \frac{q\delta}{A_1};\ \ p_{14} = \frac{\lambda}{A_1} \text{ where } A_1 = \lambda + p\delta + q\delta,$$

$$p_{26} = \frac{r}{A_2} \text{ where } A_2 = \lambda + r \quad p_{30} = \frac{\eta}{A_3}, p_{37} = \frac{\lambda}{A_3} \quad \text{where } A_3 = \lambda + \eta$$

$$p_{58} = 1, \; p_{61} = \frac{\lambda_1}{A_6}, \quad p_{68} = \frac{\lambda}{A_6} \quad \text{where } A_6 = \lambda + \lambda_1 \qquad p_{71} = 1, p_{84} = \frac{\lambda_1}{A_8}, \; p_{89} = \frac{p\delta}{A_8},$$

$$p_{95} = \frac{\lambda_1}{A_9}, p_{9,11} = \frac{r}{A_9} \text{ where } A_9 = \lambda_1 + r \; p_{10,6} = \frac{\eta}{A_{10}}, p_{10,7} = \frac{\lambda_1}{A_{10}} \quad \text{where } A_{10} = (\lambda_1 + \eta)$$

$$p_{11,1} = 1, \; p_{12,8} = 1 \tag{1}$$

Sojourn Times

Mean sojourn time μ_i in S_i is defined as the time that the system continues in state S_i before transiting to any other state are-

$$\mu_0 = \frac{1}{2\lambda} = \frac{1}{A_0}, \quad \mu_1 = \frac{1}{A_1}, \quad \mu_2 = \frac{1}{A_2}, \mu_3 = \frac{1}{A_3} \; \mu_4 = \frac{1}{A_4} \quad \mu_5 = \frac{1}{A_5}, \quad \mu_6 = \frac{1}{A_6}$$

$$\mu_7 = \frac{1}{A_7}, \; \mu_8 = \frac{1}{A_8}, \; \mu_9 = \frac{1}{A_{9\,9}} \; \mu_{10} = \frac{1}{A_{10}}, \mu_{11} = \frac{1}{A_{11}} \tag{2}$$

Mean Time to System Failure:

Let T_i be the random variable depicting time to system failure when system starts from state $S_i \in E(i = 0-3,6,8-11)$ and $\pi_i(t) = P[T_i \le t]$.To calculate the distribution function $\pi_i(t)$, we regard the failed states $S_4 S_5$ and S_7 as absorbing states. To obtain $\pi_0(t)$, we consider the possible transitions from S_0. Thus

$$\begin{aligned}
\pi_0(t) &= Q_{01}(t)[s]\; \pi_1(t) \\
\pi_1(t) &= Q_{12}(t)[s]\pi_2(t) + Q_{13}(t)[s]\pi_3(t) + Q_{14}(t) \\
\pi_2(t) &= Q_{25}(t) + Q_{26}(t)[s]\; \pi_6(t) \\
\pi_3(t) &= Q_{30}(t)[s]\pi_0(t) + Q_{37}(t) \\
\pi_6(t) &= Q_{61}(t)[s]\; \pi_1(t) + Q_{68}(t)[s]\; \pi_8(t) \\
\pi_8(t) &= Q_{84}(t) + Q_{89}(t)[s]\; \pi_9(t) + Q_{8,10}(t)[s]\pi_{10}(t) \\
\pi_9(t) &= Q_{95}(t) + Q_{9,11}(t)[s]\pi_{11}(t) \\
\pi_{10}(t) &= Q_{10,6}(t)[s]\; \pi_6 + Q_{10,7}(t) \\
\pi_{11}(t) &= Q_{11,8}(t)[s]\; \pi_8(t)
\end{aligned} \tag{3}$$

Taking Laplace-Stieltjes transforms of equations (3), the solution for $\tilde{\pi}_0(0)$, when the system starts from S_0, can be written in the following

$$E(T) = -\frac{d}{ds}\tilde{\pi}_0(s)\Big|_{s=0} = \frac{D_1'(0) - N_1'(0)}{D_1(0)} \quad [4]$$

Where

$$N_1(0) = p_{01}p_{14} - p_{01}p_{14}p_{89}p_{9,11}p_{11,8} - p_{01}p_{14}p_{68}p_{8,10}p_{10,6} + p_{01}p_{12}p_{25} - p_{01}p_{12}p_{25}p_{89}p_{9,11}p_{11,8} - p_{01}p_{12}p_{25}p_{68}p_{8,10}p_{10,6} + p_{01}p_{12}p_{26}p_{68}p_{84} + p_{01}p_{12}p_{26}p_{68}p_{89}p_{95} + p_{01}p_{12}p_{26}p_{68}p_{8,10}p_{10,7} + p_{01}p_{13}p_{37} - p_{01}p_{13}p_{37}p_{89}p_{9,11}p_{11,8} - p_{01}p_{13}p_{37}p_{68}p_{8,10}p_{10,6}$$

$$D_1(0) = 1 - p_{89}p_{9,11}p_{11,8} - p_{68}p_{8,10}p_{10,6} - p_{12}p_{26}p_{61} + p_{12}p_{26}p_{61}p_{89}p_{9,11}p_{11,8} - p_{01}p_{13}p_{30} + p_{01}p_{13}p_{30}p_{89}p_{9,11}p_{11,8} + p_{01}p_{13}p_{30}p_{68}p_{8,10}p_{10,6}$$

$$D'_1(0) - N'_1(0) = (1 - p_{12}p_{26}p_{61})(1 - p_{89}p_{9,11} - p_{8,10}p_{10,6})\mu_0 + (1 - p_{89}p_{9,11} - p_{68}p_{8,10}p_{10,6})(\mu_1 + p_{12}\mu_2 + p_{13}\mu_3) + p_{12}p_{26}(1 - p_{89}p_{9,11})\mu_6 + p_{12}p_{26}p_{68}(\mu_8 + p_{89}\mu_9 + p_{8,10}\mu_{10} + p_{89}p_{9,11}\mu_{11})$$

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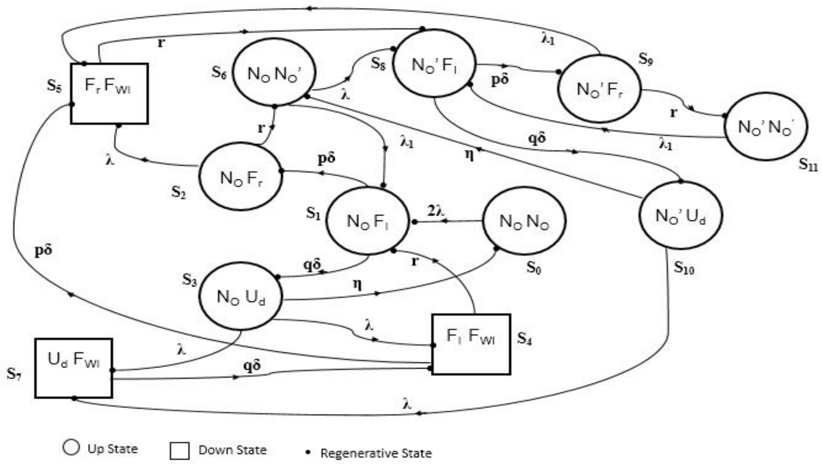
APPENDICES



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S0
NO' Ud
S10
S3
NO Ud
FI FWI
S4
S7
Ud FWI
Up State
Down State
Regenerative State